**Math 201**
**Practice questions for Quiz#2**
**To the Student: Following are practice exercises that you should do on your own, in order to prepare for Quiz 2. I emphasize, you should solve these on your own without soliciting help.Methods of solution have been covered in lectures and in recitations. Needless to say, the number of questions on the quiz will be less.**

1. Two curves $C$ and $D$ are given in polar coordinates by the equations $r = c(1-\cos\theta)$ and $r = c\cos\theta$ where $c > 0$.Find all points of intersection, and sketch the curves.

   (a) Find with justification $\lim_{(x,y)\to(0,0)}(2x^2+3y^2)\sin(\frac{1}{2x^2+3y^2})$.

   (b) Does $\lim_{(x,y)\to(0,0)} \frac{x^2y^4}{x^4+y^8}$ exist? Justify your answer. Does $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+\sin^2 y}$ exist? Justify your answer.

   (c) Let $f(x,y) = \frac{x^3y^3}{x^2+y^2}, (x,y) \neq (0,0)$ and $f(0,0) = 0$. Find both $f_x(0,0), f_y(0,0)$. Is $f$ differentiable at $(0,0)$? Prove your answer.

   (d) Let $w = \ln(2x+y^2+z^2)$.Find $\nabla w \cdot \nabla w$. Now suppose $x = 2st, y = s-t, z = s+t$.Compute $\frac{\partial w}{\partial t}$ and simplify your answer as far as possible.

2. Let $f(x,y) = x^2+y^2+3x-4y+5$ where $(x,y)$ belongs to the domain $D = \{(x,y) : x^2+y^2 \leq 2, x \geq 0\}$.Find the maximum and minimum values of $f$ on $D$.

3. Let $f$ be the function of two variables given by .

$$f(x,y) = x^2+y^2+2x-4y+1$$

   and defined on the semi-circular region

$$D = \{(x,y) : x^2+y^2 \leq 9, 0 \leq y \leq 3\}.$$

   Find, with justification, the **maximum** and **minimum** values of $f$ on $D$.

4. Find parametric equations of the tangent line to the curve of intersection of the two surfaces $x^2+y^2+z^2 = 16$ and $x^2+y^2-4y = 0$, at the point $P(2,2,2\sqrt{2})$.

5. Find an equation of the plane tangent to the surface $x^2+y^2-4y = 0$ at the point $P(2,2,\sqrt{8})$.

6. The directional derivative of $f(x,y,z)$ at a point $P$ is greatest in the direction of $\mathbf{v} = \mathbf{i}+\mathbf{j}-\mathbf{k}$. In this direction, the value of the derivative is $2\sqrt{3}$. Find $\nabla f(P)$, and then find the directional derivative of $f$ at $P$ in the direction of $\mathbf{i}+\mathbf{j}$.

7. Let $S = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$. If we use $S_{10}$ as an approximation for $S$, how large can the error be?

8. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$. If we use $S_{10}$ as an approximation for $S$, how large can the error be?

9. Find an approximation of

$$\int_0^{0.2} e^{-5x^2} dx$$

with an error that does not exceed $10^{-4}$, by doing the following steps:

- (a) Write down the Maclaurin series of $e^{(u)}$.
- (b) Use (a) to write down the Maclaurin series for $e^{-5x^2}$.
- (c) Use (b) to write down a series for $\int_0^{0.2} e^{-5x^2} dx$
- (d) Decide how many terms you want to use from (c) to get an approximation of $\int_0^{0.2} e^{-5x^2} dx$ with error less than $10^{-4}$.